



B.K. BIRLA CENTRE FOR EDUCATION

SARALA BIRLA GROUP OF SCHOOLS
A CBSE DAY-CUM-BOYS' RESIDENTIAL SCHOOL



PRE MID TERM EXAMINATION 2025-26 MATHEMATICS (Marking Scheme)

Class: XII A
Date: 06/08/25
Admission no:

Time: 1hr
Max Marks: 25
Roll no:

General Instructions:

Question 1 to 5 carries ONE mark each. Questions 6 to 9 carries TWO marks each.
Questions 10 to 13 carries THREE marks each.

1. The function $f(x) = \frac{4-x^2}{4x-x^3}$
(A) Discontinuous at only one point
(B) Discontinuous exactly at two points.
(C) **Discontinuous at exactly three points.**
(D) None of these
2. If $x = \cos\theta$, $y = \sin\theta$, then $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$.
(A) 1 (B) 0 (C) **-1** (D) None of these
3. If $y = 1 + e^{3x}$, find $\frac{d^2y}{dx^2}$
(A) 0 (B) $6e^{3x}$ (C) **$9e^{3x}$** (D) None of these
4. If $x = t^2$ and $y = t^3$ then $\frac{d^2y}{dx^2}$
(A) **$\frac{3}{2}$** (B) $\frac{3}{4}t$ (C) $\frac{3}{2}t$ (D) $\frac{3}{4}$

Assertion and Reasoning questions: In the following two questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

- (A) Both A and R are true and R is the correct explanation of A.
(B) Both A and R are true and R is not the correct explanation of A.
(C) **A is true but R is false.**
(D) A is false but R is true.
5. Assertion (A): The function $f(x) = e^{-|x|}$ continuous for all x.
Reason (R): $f(x) = e^{-|x|}$ is differentiable for all x.

- 6 Determine the value of 'k' for which the following function is continuous at

$$x=3, f(x)=\begin{cases} \frac{(x)^2-9}{x-3}, & x \neq 3 \\ k, & x = 3 \end{cases}$$

Sol: Given function is continuous at $x=3$, $\lim_{x \rightarrow 3} f(x) = f(3)$

$$\lim_{x \rightarrow 3} \frac{x^2-9}{x-3} = \lim_{x \rightarrow 3} \frac{(x+3)(x-3)}{x-3} = \lim_{x \rightarrow 3} x + 3 = 6 = f(3) = k = 6$$

- 7 If $xy = 1$, Prove that $\frac{dy}{dx} + y^2 = 0$

Sol: Apply product rule 1

$$dy/dx + y^2 = 0$$

- 8 Differentiate $(\sin x)^{\cos x}$ with respect to x .

Sol: $y = (\sin x)^{\cos x}$ apply log to both side $\log y = \cos x \log \sin x$

Diff. wrt x : $1/y \frac{dy}{dx} = -\sin x \log \sin x + \cos x (1/\sin x) \cos x$

$$= y(-\sin x \log \sin x + \cos x \cot x)$$

$$= (\sin x)^{\cos x} (-\sin x \log \sin x + \cos x \cot x)$$

- 9 The volume of a cube is increasing at a constant rate. Prove that the increase in Surface area varies inversely as the length of the edge of the cube.

Sol: $S = 6x^2$ and $V = x^3$ and it is given that $\frac{dv}{dx} = k$

$$dx/dt = k/3x^2$$

$$S = 6x^2, ds/dt = 12x dx/dt, ds/dt = 12x \cdot k/3x^2 = 4k/x$$

$$ds/dt \propto 1/x$$

- 10 If the function $f(x)$, given by $f(x) = \begin{cases} 3ax + b, & \text{if } x > 1 \\ 11, & \text{if } x = 1 \\ 5ax - 2b, & \text{if } x < 1 \end{cases}$ is continuous at $x=1$.

Sol: (LHL at $x=1$) = $5a-2b$

(RHL at $x=1$) = $3a+b$ and $f(1) = 11$, since $f(x)$ is continuous at $x=1$

$$\text{LHL at } x=1 = \text{RHL at } x=1 = f(1)$$

$$5a-2b = 3a+b = 11$$

$$5a-2b = 11 \text{ and } 3a+b=11, a=3 \text{ and } b=2.$$

- 11 If $x^2 + 2xy + y^2 = 42$, find $\frac{dy}{dx}$.

Sol: $2x + 2(xdy/dx + y) + 3y^2 dy/dx = 0$

$$2x + 2y = -(2x + 3y^2) dy/dx$$

$$\frac{dy}{dx} = \frac{2x+2y}{2x+3y^2}$$

- 12 If $y = \sin^{-1}x$, show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$.

Sol: $y = \sin^{-1}x$, differentiate wrt x , $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$, $\sqrt{1-x^2}\frac{dy}{dx} = 1$

Differentiate again wrt x $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$

- 13 Find the intervals in the function $f(x) = 2x^3 + 9x^2 + 12x + 20$, (i) increasing, (ii) decreasing.

Sol: $f'(x) = 6x^2 + 18x + 12 = 0$

$x = -1$ and -2

Now, check the behaviour of derivative on interval

$(-\infty, -2)$, $(-2, -1)$ and $(-1, \infty)$

We found f is increasing on $(-\infty, -2) \cup (-1, \infty)$

and f is decreasing on $(-2, -1)$
